Unit 7: Multivariate Analysis

Statistics for Linguists with R – A SIGIL Course

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Outline

Introduction
- Multivariate analysis
- Setting up

Mathematical background
- Feature matrix
- Distance metric
- Orthogonal projection

Outline

What is multivariate analysis?

- Univariate statistics
  - focus on a single variable of interest (at a time)
  - estimate population parameters (π, μ, σ^2, ...)
  - comparison of two or more groups

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  - based on multiple other variables ("independent")

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  - focus on interdependencies of two variables
  - correlation & co-occurrence
- Regression modelling
  - predict single target variable ("dependent")
  - based on multiple other variables ("independent")
- Multivariate statistics
  - combined effects of many variables
  - correlations & distribution patterns
  - often "unsupervised": no target variable or comparison groups

Application examples

- Register variation (Biber 1988, 1993)
- Translation studies
  (Evert & Neumann 2017; De Sutter *et al.* 2012)
- Stylometry: authorshop attribution (Evert *et al.* 2017)
- Dialectology (Speelman *et al.* 2003)
- Historical linguistics (Sagi *et al.* 2009; Perek 2018)
- Identification of confounding variables (Tummers *et al.* 2014)
- Linguistic productivity (Jenset & McGillivray 2012)
- Correspondence analysis (Greenacre 2007)
- Distributional semantics (see ESSLLI course)

Outline

R packages

Required R packages:

- `corpora` ($\geq$ 0.5)
- `wordspace` ($\geq$ 0.2)

Recommended packages:

- `ggplot2`, `reshape2` … for plotting feature weights
- `rgl` … for interactive 3-d visualization
- `Hotelling`, `ellipse` … for significance testing
- `e1071` … for machine learning (SVM)
- `Rtsne` … for low-dimensional maps
- `ca` … for correspondence analysis

☞ install with package manager in RStudio or R GUI

Code & data sets

Download additional code & data sets from SIGIL homepage:

- multivar_utils.R
- unit7_data.rda

☞ put all files in RStudio project directory (or working directory)

```
> library(corpora)               # basic utilities and some data sets
> library(wordspace)             # for large and sparse matrices

> source("multivar_utils.R")     # additional functions

> load("unit7_data.rda", verbose=TRUE)   # further data sets
```

Overview of data sets

- 65 Biber features for British National Corpus
  - BNCbiber = 4048 × 65 feature matrix
  - BNCmeta = complete metadata table
  - extensive documentation with ?BNCbiber, ?BNCmeta
- 67 Biber features for Brown Family corpora
  - BrownBiber_Matrix = 3500x67 feature matrix
  - BrownBiber_Meta = metadata table
  - features are Biber-scaled z-scores obtained with MAT v1.3
    http://sites.google.com/site/multidimensionaltagger/
  - see tagger manual for feature definitions

Overview of data sets

- 27 SFL-inspired features for translation pairs (CroCo corpus)
  - `CroCo_Matrix` = 452 × 27 feature matrix
  - `CroCo_Meta` = metadata table
  - `CroCo_orig2trans` = row numbers of translation pairs
  - data from Evert & Neumann (2017)
- Literary authorship attribution with Δ measures
  - data: sparse document-term matrices for 20,000 most frequent words (mfw) as `wordspace` DSM objects
  - `Delta$DE` = 75 × 20000 matrix (German novels, 25 authors)
  - `Delta$EN` = 75 × 20000 matrix (English novels, 25 authors)
  - `Delta$FR` = 75 × 20000 matrix (French novels, 25 authors)
  - `Delta$DE$rows`, `Delta$EN$rows`, ... = metadata tables
  - `DeltaLemma` = lemmatized version
  - data from Jannidis *et al.* (2015); Evert *et al.* (2017)

Overview of data sets

- 19 type-token complexity measures for Δ corpus
  - complexity scores for 10,000-token text slices from 75 novels
  - `DeltaComplexity$DE$Matrix` = 996×19 matrix (German)
  - `DeltaComplexity$EN$Matrix` = 1147×19 matrix (English)
  - `DeltaComplexity$FR$Matrix` = 679×19 matrix (French)
  - `DeltaComplexity$DE$Meta`, … = metadata tables
  - can be used to study correlational patterns between measures
- 7 syntactic complexity measures for 969 German novels
  - `SyntacticComplexity_Matrix` = 969×7 feature matrix
  - `SyntacticComplexity_Meta` = metadata tables
  - can be used to compare high-brow against low-brow literature

Outline

Feature matrix

Feature matrix records quantitative features for each text

$$\mathbf{M} = \begin{bmatrix} \cdots & \mathbf{m}_1 & \cdots \\ \cdots & \mathbf{m}_2 & \cdots \\ & \vdots & \\ & \vdots & \\ \cdots & \mathbf{m}_k & \cdots \end{bmatrix}$$

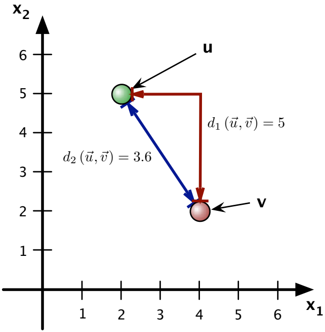
	nominal	pass	prep	subord	ttr
$orig_1$	1.205	5.013	6.883	4.483	1.285
$orig_2$	0.738	2.537	6.486	6.157	1.714
$orig_3$	1.252	4.462	8.463	4.785	2.476
$orig_4$	1.105	2.899	8.119	3.966	1.519
$orig_5$	1.764	4.268	7.167	3.947	1.792
$orig_8$	1.545	7.268	7.461	5.455	1.572
$trans_1$	0.463	2.208	6.297	6.089	2.339
$trans_2$	1.131	2.597	6.307	4.844	1.810
$trans_4$	0.935	1.744	7.098	4.012	1.403
$trans_5$	0.867	3.604	7.511	5.154	1.902
$trans_7$	1.387	4.290	8.211	3.998	1.822

```
> M <- MultiVar_Matrix
> M
```

Outline

Geometric distance = metric

- **Distance** between vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ → (dis)similarity
  - $\mathbf{u} = (u_1, \ldots, u_n)$
  - $\mathbf{v} = (v_1, \ldots, v_n)$



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  - $\mathbf{u} = (u_1, \ldots, u_n)$
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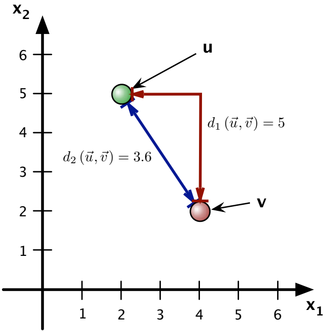
- **Euclidean** distance $d_2(\mathbf{u}, \mathbf{v})$



$$d_2(\mathbf{u}, \mathbf{v}) := \sqrt{(u_1 - v_1)^2 + \cdots + (u_n - v_n)^2}$$

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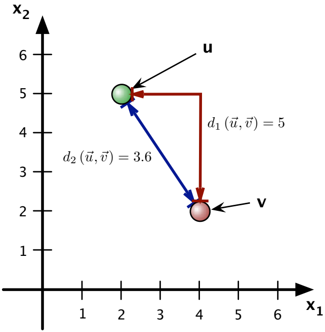
- "City block" **Manhattan** distance $d_1(\mathbf{u}, \mathbf{v})$



$$d_1(\mathbf{u}, \mathbf{v}) := |u_1 - v_1| + \cdots + |u_n - v_n|$$

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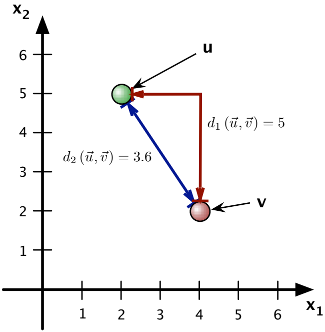
- **Euclidean** distance $d_2(\mathbf{u}, \mathbf{v})$
- "City block" **Manhattan** distance $d_1(\mathbf{u}, \mathbf{v})$
- Both are special cases of the **Minkowski** p-distance $d_p(\mathbf{u}, \mathbf{v})$ (for $p \in [1, \infty]$)



$$d_p(\mathbf{u}, \mathbf{v}) := \left(|u_1 - v_1|^p + \cdots + |u_n - v_n|^p \right)^{1/p}$$

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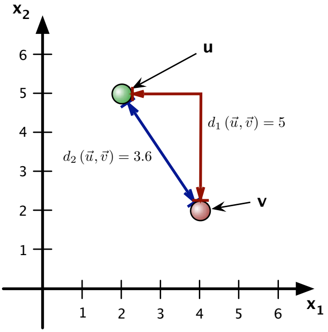

$$d_p(\mathbf{u}, \mathbf{v}) := \left(|u_1 - v_1|^p + \cdots + |u_n - v_n|^p\right)^{1/p}$$

$$d_\infty(\mathbf{u}, \mathbf{v}) = \max\left\{|u_1 - v_1|, \ldots, |u_n - v_n|\right\}$$

Geometric distance = metric

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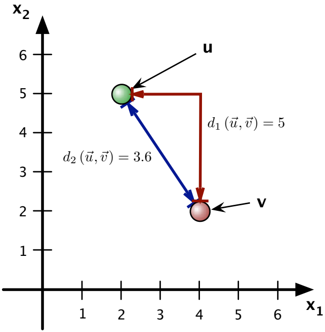
  - $\mathbf{u} = (u_1, \ldots, u_n)$
  - $\mathbf{v} = (v_1, \ldots, v_n)$
- **Euclidean** distance $d_2(\mathbf{u}, \mathbf{v})$
- "City block" **Manhattan** distance $d_1(\mathbf{u}, \mathbf{v})$
- Extension of p-distance $d_p(\mathbf{u}, \mathbf{v})$ (for $0 \le p \le 1$)

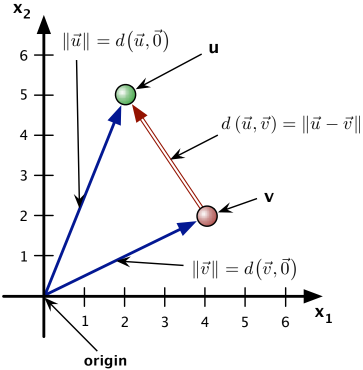


$$d_p(\mathbf{u}, \mathbf{v}) := |u_1 - v_1|^p + \cdots + |u_n - v_n|^p$$

$$d_0(\mathbf{u}, \mathbf{v}) = \#\{i \mid u_i \neq v_i\}$$

Distance and vector length = norm

- Intuitively, distance $d(\mathbf{u}, \mathbf{v})$ should correspond to length $\|\mathbf{u} - \mathbf{v}\|$ of displacement vector $\mathbf{u} - \mathbf{v}$
  - $d(\mathbf{u}, \mathbf{v})$ is a **metric**
  - $\|\mathbf{u} - \mathbf{v}\|$ is a **norm**
  - $\|\mathbf{u}\| = d(\mathbf{u}, \mathbf{0})$



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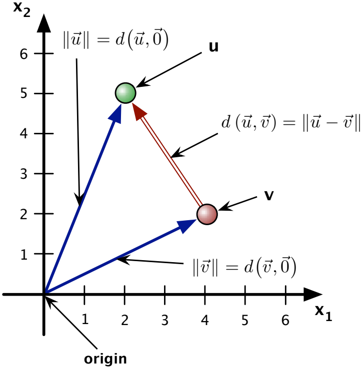

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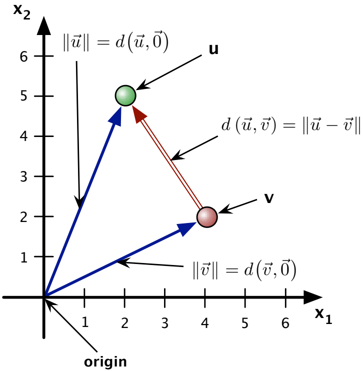

- $d_p(\mathbf{u}, \mathbf{v}) = \|\mathbf{u} - \mathbf{v}\|_p$
- **Minkowski p-norm** for $p \in [1, \infty]$ (not $p < 1$):

$$\|\mathbf{u}\|_p := \left(|u_1|^p + \cdots + |u_n|^p\right)^{1/p}$$

Computing distances

Compute distances between all pairs of texts:

```
> round(dist(M), 2)    # returns a triangular 'dist' object
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Use `wordspace` function for additional metrics:

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> dist.matrix(M, method="mink", p=0.5)    # full matrix
> dist.matrix(M, method="mink", p=0.5, as.dist=TRUE)
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```

Standardize features for equal contribution to Euclidean metric:

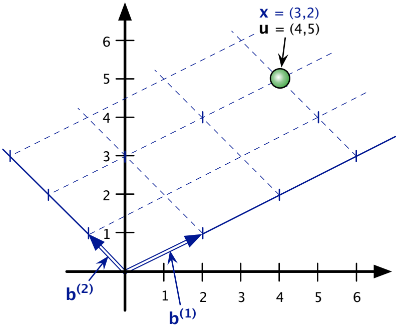
```
> Z <- scale(M)       # matrix of z-scores
> round(dist(Z), 2) # default: Euclidean metric
```

Outline

Linear subspace & basis

- A linear **subspace** $B \subseteq \mathbb{R}^n$ of rank $r \leq n$ is spanned by a set of r linearly independent basis vectors

$$B = \{\mathbf{b}_1, \ldots, \mathbf{b}_r\}$$



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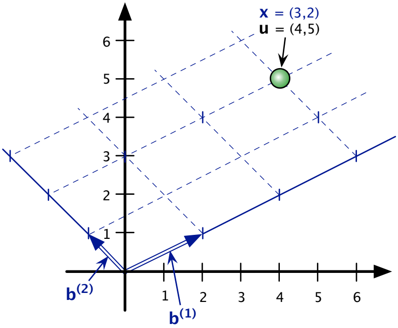
- Every point $\mathbf{u}$ in the subspace is a unique linear combination of the basis vectors

$$\mathbf{u} = x_1\mathbf{b}_1 + \ldots + x_r\mathbf{b}_r$$

- Coordinate vector $\mathbf{x} \in \mathbb{R}^r$ with respect to the basis



Linear subspace & basis

- Basis matrix $\mathbf{V} \in \mathbb{R}^{n \times r}$ with column vectors $\mathbf{b}_i$:

$$\mathbf{u} = x_1 \mathbf{b}_1 + \ldots + x_r \mathbf{b}_r = \mathbf{V}\mathbf{x}$$

$$\begin{bmatrix} x_1 b_{11} + \ldots + x_r b_{1r} \\ x_1 b_{21} + \ldots + x_r b_{2r} \\ \vdots \\ x_1 b_{n1} + \ldots + x_r b_{nr} \end{bmatrix} = \begin{bmatrix} b_{11} & \cdots & b_{1r} \\ b_{21} & \cdots & b_{2r} \\ \vdots & & \vdots \\ b_{n1} & \cdots & b_{nr} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_r \end{bmatrix}$$

$$\underset{(n \times 1)}{\mathbf{u}} = \underset{(n \times r)}{\mathbf{V}} \cdot \underset{(r \times 1)}{\mathbf{x}}$$

An aside: Matrix multiplication

$$\begin{bmatrix} & a_{ij} & \\ & & \\ & & \end{bmatrix} = \begin{bmatrix} b_{i1} & \cdots & b_{in} \\ & & \\ & & \end{bmatrix} \cdot \begin{bmatrix} & c_{1j} & \\ & \vdots & \\ & \vdots & \\ & c_{nj} & \end{bmatrix}$$

$$\underset{(k \times m)}{\mathbf{A}} = \underset{(k \times n)}{\mathbf{B}} \cdot \underset{(n \times m)}{\mathbf{C}}$$

- **B** and **C** must be **conformable** (in dimension n)
- Element a_{ij} is the inner product of the i-th row of **B** and the j-th column of **C**

$$a_{ij} = b_{i1}c_{1j} + \ldots + b_{in}c_{nj} = \sum_{t=1}^{n} b_{it}c_{tj}$$

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Orthonormal basis

- Particularly convenient with orthonormal basis:

$$
\begin{aligned}
\|\mathbf{b}_i\|_2 &= 1 \\
\mathbf{b}_i^T \mathbf{b}_j &= 0 \qquad\qquad \text{for } i \neq j
\end{aligned}
$$

- Corresponding basis matrix $\mathbf{V}$ is (column)-**orthogonal**

$$\mathbf{V}^T \mathbf{V} = \mathbf{I}_r$$

and defines a Cartesian coordinate system in the subspace

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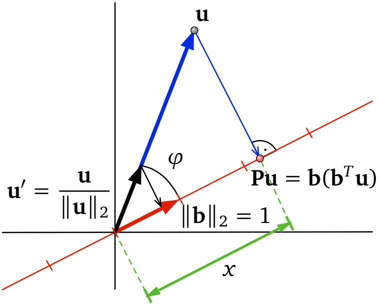
☞ From now on always assume orthonormal basis

The mathematics of projections

- 1-d subspace spanned by basis vector $\|\mathbf{b}\|_2 = 1$
- For any point $\mathbf{u}$, we have

$$\cos\varphi = \frac{\mathbf{b}^T\mathbf{u}}{\|\mathbf{b}\|_2 \cdot \|\mathbf{u}\|_2} = \frac{\mathbf{b}^T\mathbf{u}}{\|\mathbf{u}\|_2}$$



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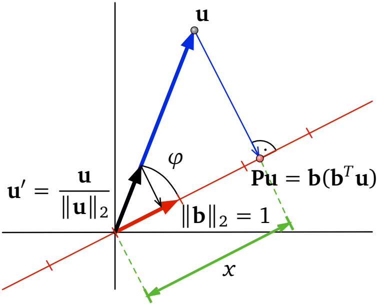
- Trigonometry: coordinate of point on the line is $x = \|\mathbf{u}\|_2 \cdot \cos\varphi = \mathbf{b}^T\mathbf{u}$



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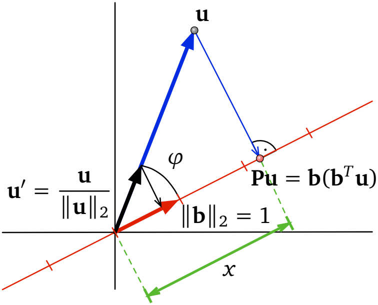

- The projected point in original space is then given by

$$\mathbf{b} \cdot x = \mathbf{b}(\mathbf{b}^T\mathbf{u}) = (\mathbf{b}\mathbf{b}^T)\mathbf{u} = \mathbf{P}\mathbf{u}$$

where $\mathbf{P}$ is a **projection matrix** of rank 1

The mathematics of projections

- For an orthogonal basis matrix $\mathbf{V}$ with columns $\mathbf{b}_1, \dots, \mathbf{b}_r$, the projection into the rank-r subspace B is given by

$$\mathbf{P}\mathbf{u} = \left(\sum_{i=1}^{r} \mathbf{b}_i \mathbf{b}_i^T \right) \mathbf{u} = \mathbf{V}\mathbf{V}^T \mathbf{u}$$

  and its subspace coordinates are $\mathbf{x} = \mathbf{V}^T \mathbf{u}$

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- Projection can be seen as decomposition into the projected vector and its orthogonal complement

$$\mathbf{u} = \mathbf{P}\mathbf{u} + (\mathbf{u} - \mathbf{P}\mathbf{u}) = \mathbf{P}\mathbf{u} + (\mathbf{I} - \mathbf{P})\mathbf{u} = \mathbf{P}\mathbf{u} + \mathbf{Q}\mathbf{u}$$

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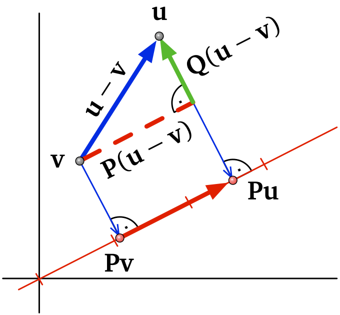
- Because of orthogonality, this also applies to the squared Euclidean norm (according to the Pythagorean theorem)

$$\|\mathbf{u}\|^2 = \|\mathbf{P}\mathbf{u}\|^2 + \|\mathbf{Q}\mathbf{u}\|^2$$

Optimal projections and subspaces

- Orthogonal decomposition of squared distances btw. vectors

$$\|\mathbf{u}-\mathbf{v}\|^2 = \|\mathbf{Pu}-\mathbf{Pv}\|^2 + \|\mathbf{Qu}-\mathbf{Qv}\|^2$$



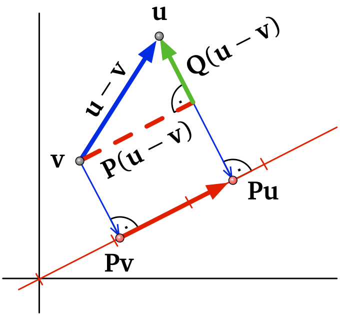
Optimal projections and subspaces

- Orthogonal decomposition of squared distances btw. vectors

$$\|\mathbf{u}-\mathbf{v}\|^2 = \|\mathbf{Pu}-\mathbf{Pv}\|^2 + \|\mathbf{Qu}-\mathbf{Qv}\|^2$$

- Define projection **loss** as difference btw. squared distances

$$\big|\, \|\mathbf{P}(\mathbf{u}-\mathbf{v})\|^2 - \|\mathbf{u}-\mathbf{v}\|^2 \,\big|$$
$$= \|\mathbf{u}-\mathbf{v}\|^2 - \|\mathbf{P}(\mathbf{u}-\mathbf{v})\|^2$$
$$= \|\mathbf{Q}(\mathbf{u}-\mathbf{v})\|^2$$



Optimal projections and subspaces

- Orthogonal decomposition of squared distances btw. vectors

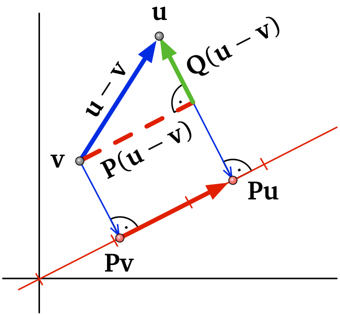
$$\|\mathbf{u}-\mathbf{v}\|^2 = \|\mathbf{P}\mathbf{u}-\mathbf{P}\mathbf{v}\|^2 + \|\mathbf{Q}\mathbf{u}-\mathbf{Q}\mathbf{v}\|^2$$

- Define projection **loss** as difference btw. squared distances

$$\begin{aligned} &\big|\,\|\mathbf{P}(\mathbf{u}-\mathbf{v})\|^2 - \|\mathbf{u}-\mathbf{v}\|^2\,\big| \\ &= \|\mathbf{u}-\mathbf{v}\|^2 - \|\mathbf{P}(\mathbf{u}-\mathbf{v})\|^2 \\ &= \|\mathbf{Q}(\mathbf{u}-\mathbf{v})\|^2 \end{aligned}$$

- Projection quality measure:

$$R^2 = \frac{\|\mathbf{P}(\mathbf{u}-\mathbf{v})\|^2}{\|\mathbf{u}-\mathbf{v}\|^2}$$



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